JOURNAL OF MATERIALS SCIENCE **32** (1997) 6513–6517

# On crack stability in paper toughness testing

YONGZHONG YU, PETRI KÄRENLAMPI
*Department of Forest Resource Management, Box 24, FIN-00014 Univeristy of Helsinki, Finland*

The elastic energy stored in a double-edged notched tension specimen at the moment of crack propagation initiation is analysed. The stored energy increases linearly with specimen length and, in the range of interest, almost linearly with crack length. Experiments show somewhat surprisingly that copy paper specimen dimensions contribute to the external fracture work very much according to the linear elastic analysis. The specific essential fracture work of such a material cannot be determined as proposed by Cotterell and Reddel, this obviously applying to most paper grades. The fracture work of extremely tough and ductile sack paper appears to be independent of specimen dimensions as long as the sample is not longer than its width.

## 1. Introduction

The specific essential fracture work of ductile materials [1, 2] can be measured as proposed by Cotterell and Reddel [3]. For given thickness, the specific essential work of fracture of a few materials has been shown to be independent of specimen geometry [4, 5, 6], which is a requirement for a material property. However, only some geometries make it possible to separate the essential and non-essential work of crack propagation. One of these constructions is the double-edge notched tension specimen (DENT), where the area of plastic deformations is semicircular, and thus the work consumed for plastic deformations outside the outer plastic zone is proportional to the square of the length of the ligament [3]. This yields the total fracture work.

$$W_f = Ltw_f = Ltw_e + bL^2tw_p \tag{1}$$

where $L$ is the length of the ligament to be torn, $t$ is thickness, $w_f$ is the specific total fracture work, $w_e$ is the specific essential fracture work, $b$ is a geometrical shape parameter of the outer plastic zone and $w_p$ is the specific non-essential fracture work. Equation 1 readily implies that the specific essential work of fracture $w_e$ can be produced by extrapolating the value of $w_f$ to zero ligament length from a test series with a few ligament lengths. The method (essential work of fracture, EWF) has been applied to metals, polymers and ductile paper grades like copy and sack paper and pulp handsheets [7–13].

Since the total fracture work $W_f$ is approximated as the external work done by straining the specimen, the test requires stable cracking: all the external work done must be consumed by plastic deformations. It may be questionable to which degree this is possible. Crack stability analysis by Mai [14] proposes that for a linearly elastic material, the DENT geometry is inherently unstable. According to Broberg [2, 15] in tough, ductile materials the plastic region screens the energy flow to the essential fracture process zone, and thus stable crack growth usually precedes catastrophic failure. However, no exact stability criterion can be given for plasticizing materials.

Very pronounced instability can be detected as a sudden drop in the load–elongation curve. However, in many cases it is difficult to know if the crack really has been stable. Let us consider what happens in the fracture toughness test if instability occurs and the external work done is greater than the energy consumed by plastic deformations.

If the ratio or the absolute difference of the external work to $W_f$ is roughly constant, an overestimation of $w_e$ takes place. The extent of this overestimation depends on the degree of instability, which in turn relates to material ductility. This results in biased comparisons in the fracture toughness values of different materials. It is further possible that the eventual instability is not independent of specimen dimensions [cf. 14]. Thus it is possible that the ratio of external work to fracture work changes with ligament length, which may result as an overestimation or underestimation of $w_e$.

Recent observations by the authors suggest that the estimate of the specific essential fracture work of copy paper and pulp handsheets is not necessarily independent of specimen dimensions. In this paper, we intend to clarify if crack stability can possibly be achieved by appropriate specimen design. First, we present a linear elastic fracture mechanics analysis of the effect of specimen dimensions on the ratio of elastic strain energy stored in the specimen at the moment of crack propagation initiation to the energy consumed by crack propagation. Then we present experimental observations on the eventual effect of copy paper and sack paper specimen dimensions on the specific external work required to break the specimen.

## 2. The effect of specimen dimensions on the elastic strain energy

As long as no crack propagation has occurred, the external work $W$ done on a linear elastic structure equals the strain energy $U$

$$W = U = \tfrac{1}{2}\Delta P \tag{2}$$

where $\Delta$ is displacement and $P$ is load. Defining compliance $C$ as

$$C = \frac{\Delta}{P} \tag{3}$$

Equation 2 may be written

$$W = U = \tfrac{1}{2}CP^2 \tag{4}$$

Then the energy release rate, defined as the change of potential energy with crack area increment d$A$, can be expressed as [cf. 16]

$$G = \frac{\mathrm{d}U}{\mathrm{d}A} = \frac{P^2}{2t}\frac{\mathrm{d}C}{\mathrm{d}a} \tag{5}$$

where $t$ is specimen thickness and d$a$ is crack length increment.

For plane stress, the stress intensity factor $K$, characterizing the singularity of elastic stresses around a sharp crack, and the energy release rate $G$ have the relation

$$G = \frac{K^2}{E} \tag{6}$$

where $E$ is Young's modulus of the material. The stress intensity factor can further be given by

$$K = \beta\sigma a^{1/2} \tag{7}$$

where $\beta$ is a factor depending on specimen geometry and $\sigma$ is remote stress far from the crack.

Combining Equations 5, 6 and 7, noting that $P = A_r\sigma$, where $A_r$ is the remote cross-sectional area of the specimen, and integrating, the compliance becomes

$$C = \frac{2t}{A_r^2 E}\int_0^a \beta^2 a\,\mathrm{d}a + C_0 \tag{8}$$

where $C_0$ is the compliance at zero crack length. The external work done up to the moment of crack propagation initiation is

$$W_i = \frac{1}{2}C\frac{REA_r^2}{\beta^2 a} \tag{9}$$

Equation 9 also gives the total external work done on the DENT specimen of a linearly elastic material since the cracking is unstable, and thus the specimen fails catastrophically [14]. The work consumed by the crack propagating over a ligament of length $L$ being $RtL$, where $R$ is the fracture toughness of the material, the ratio of external work done to the work consumed by crack propagation is

$$\frac{W_i}{W_f} = \frac{CEA_r^2}{2\beta^2 atL} \tag{10}$$

For the DENT geometry with total crack length $2a$, width $2w$ and length $2h$ (Fig. 1), some solutions have

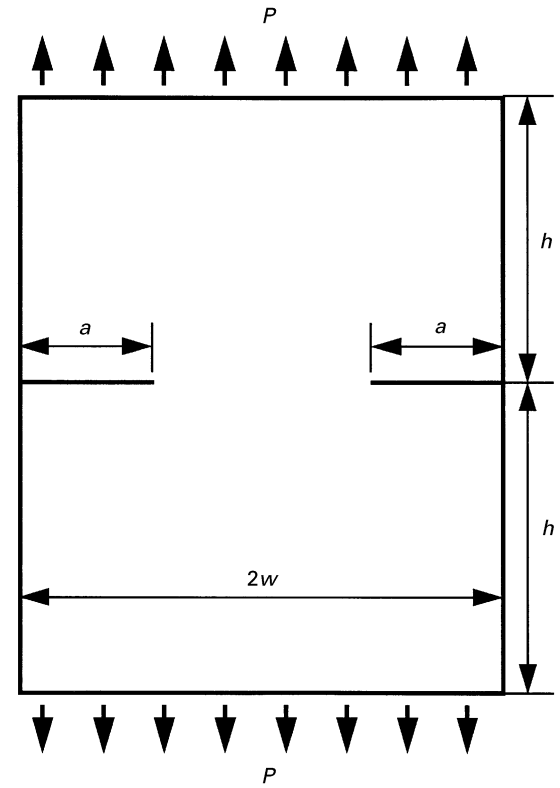

*Figure 1* Double-edged notched tension specimen geometry (DENT).

been presented for the dimensionless geometric factor $\beta$. All these solutions are approximate, but the more complicated ones [19–22] agree reasonably with the most simple one [14], which is

$$\beta = \left[\frac{2w}{a}\left(\tan\frac{\pi a}{2w} + 0.1\sin\frac{\pi a}{w}\right)\right]^{1/2} \tag{11}$$

being independent of specimen length provided that the specimen is not very short.

Assuming that the testing machine is much stiffer than the specimen to be tested, the compliance at zero crack length is

$$C_0 = \frac{\Delta_0}{P} = \frac{h}{wtE} \tag{12}$$

Combining Equations 11 and 12, Equation 8 now becomes

$$C = \frac{h}{wtE} + \frac{2}{\pi tE}\left[0.1 - 0.1\cos\frac{\pi a}{w} - 2\ln\cos\frac{\pi a}{2w}\right] \tag{13}$$

Substituting Equations 11 and 13 into Equation 9, the external work done on the DENT structure at the moment of crack propagation initiation is

$$W_i = twR\bigg/\left(\tan\frac{\pi a}{2w} + 0.1\sin\frac{\pi a}{w}\right)^{-1} \times\left(\frac{h}{w} + \frac{0.2}{\pi} - \frac{0.2}{\pi}\cos\frac{\pi a}{w} - \frac{4}{\pi}\ln\cos\frac{\pi a}{2w}\right) \tag{14}$$

Considering now that the ligament length $L$ equals $2(w - a)$, the ratio of external work to fracture work becomes

$$\frac{W_i}{W_f} = \left[2\left(1 - \frac{a}{w}\right)\left(\tan\frac{\pi a}{2w} + 0.1\sin\frac{\pi a}{w}\right)\right]^{-1} \times \left(\frac{h}{w} + \frac{0.2}{\pi} - \frac{0.2}{\pi}\cos\frac{\pi a}{w} - \frac{4}{\pi}\ln\cos\frac{\pi a}{2w}\right) \quad (15)$$

This ratio of external work to fracture work (Equation 15) is illustrated in Fig. 2. We find that this ratio is a linear function of the specimen height-to-width ratio $h/w$ and a nonlinear, non-monotonic function of the crack length-to-specimen width ratio $a/w$.

The entire range of Fig. 2, however, is not relevant for interpreting the results of the fracture toughness test. It has been recommended [3] that the specimen width-to-ligament length ratio should be at least 3, which corresponds to $a/w \geqslant 2/3$. We find from Fig. 3 that in the range of $0.60 < (a/w) < 0.85$, the ratio of external work to fracture work is an almost linear function of $a/w$, as well. We further find from Fig. 3 that the smallest value of $W_i/W_f$ achievable in this range of specimen geometry is in the order of 1.5, which means that the results of the toughness test are necessarily biased: as long as the material is linearly elastic, crack stability cannot be achieved by designing the specimen appropriately, at least not as long as $h/w \geqslant 1$.

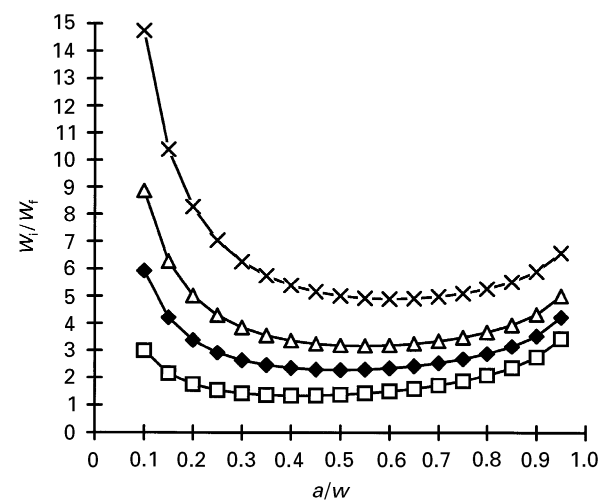


*Figure 2* The effect of the specimen dimensions on the ratio of $W_i/W_f$. (×) $h/w = 5$; (△) $h/w = 3$; (◆) $h/w = 2$; (□) $h/w = 1$.

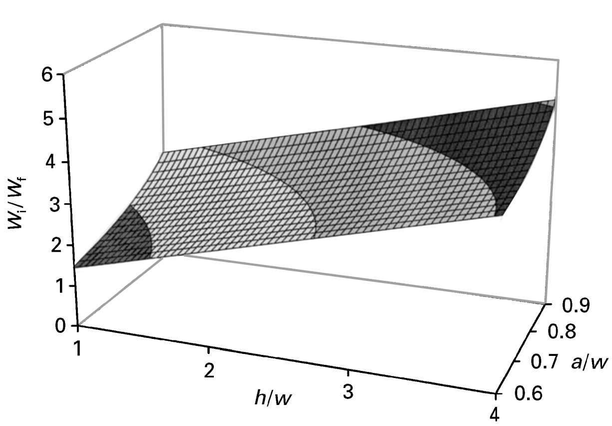


*Figure 3* The effect of a smaller range of specimen dimensions on the ratio of $W_i/W_f$. (▒) 5–6, (■) 4–5; (▒) 3–4; (□) 2–3; (■) 1–2.

Now we can analyse what happens in the EWF-test with changed specimen dimensions. Let us assume that other specimen dimensions remain constant when decreasing the ligament length, i.e. increasing crack length. $a/w$ increases, but $h/w$ remains unaffected. Thus the smaller the ligament, the greater the ratio $W_i/W_f$, and the fracture toughness of the material is thereby overestimated.

Let us then assume that we have constant specimen length but adjust specimen width in proportion with ligament length. Now the $h/w$ ratio increases strongly with decreasing ligament length, and the fracture toughness is strongly overestimated. The overestimation of fracture toughness is least if we keep both $h/w$ and $a/w$ constant with changed ligament length and minimize the values of both parameters. However, within the range illustrated in Fig. 3, the toughness is overestimated anyway and the more brittle the material, the greater the overestimation.

## 3. Experimental observations

The EWF test has not been designed for brittle materials, and thus the above arguments derived by linear elastic fracture mechanics are not applicable as such. However, it is tempting to consider to which degree some tough, ductile papers follow the above-presented elastic predictions, and to which degree the measured external fracture work is independent of specimen dimensions. For this purpose, let us consider two papers, one woodfree copy paper and another microcreped kraft sack paper. Some properties of the papers tested in the paper machine direction are given in Table I.

We find from Table I that the copy paper is stronger and stiffer than the sack paper, which in turn is extremely ductile and tough. However, even the copy paper has the ratio of the product of toughness and stiffness to the square of yield stress (0.1% offset) 50 mm [cf. 17, 18, 12].

All test pieces had the same ligament length, 15 mm, while $h/w$ varied between 0.44 and 4.0 and $a/w$ varied between 0.67 and 0.83. The samples were conditioned at 23 °C and 50% relative humidity and elongated in a stiff testing machine at $2\,\mathrm{mm\,min^{-1}}$. Regression analysis of testing data of copy paper showed that the fracture work of copy paper seems to be a linear function of $h/w$ as well as $a/w$ (Fig. 4). After explaining the fracture work with such a linear function, the residual variation is small (Fig. 4).

Fig. 5 reconfirms that the specific external work required to break the specimen clearly depends on specimen dimensions, even if the specimens are short. This demonstrates that the fracture process is not even close to being completely stable, and the specific essential fracture work cannot be determined as proposed by Cotterell and Reddel [3, cf. 9, 11, 12, 13].

A similar analysis for the sack paper is reported in Figs 6 and 7. In this case the residual variation after fitting the linear regression is not negligible. Though

TABLE I Some properties of the two experimental papers

| | Basis weight, $W$ ($g\,m^{-2}$) | Tensile stiffness, E ($N\,m\,kg^{-1}$) | Yield stress, $\sigma_\gamma$ ($N\,m\,kg^{-1}$) | Rupture strain, ε (%) | Tensile strength, $T$ ($N\,m\,g^{-1}$) | Fracture toughness, $R$ ($J\,m\,kg^{-1}$) | $RE/\sigma_\gamma^2$ (mm) |
|---|---|---|---|---|---|---|---|
| Copy paper | 80 | 8.6 | 47.6 | 1.7 | 73.7 | 13.1 | 50 |
| Sack paper | 71 | 4.2 | 21.1 | 6.4 | 60.8 | 41.7 | 390 |

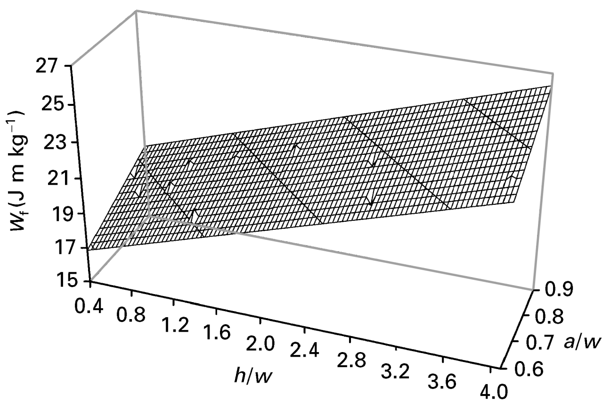


*Figure 4* External work needed to break copy paper specimens as a linear function of specimen dimensions.

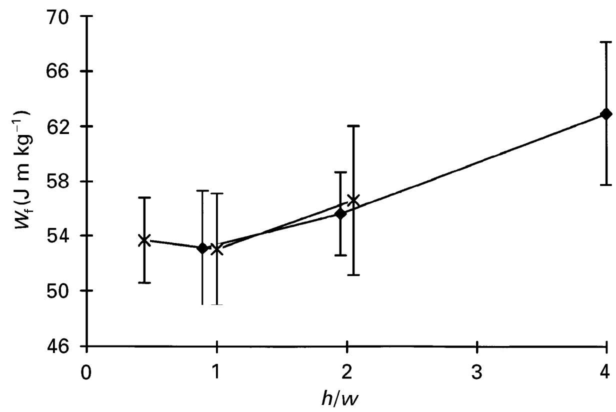


*Figure 7* The effect of specimen dimensions on the fracture work of sack paper. (*) $a/w = 0.83$; (◆) $a/w = 0.67$.

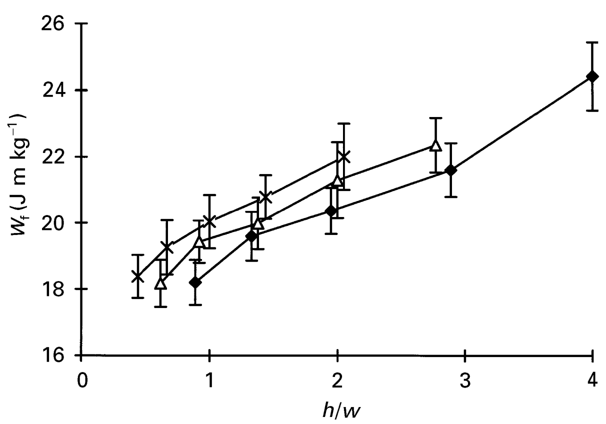


*Figure 5* The effect of specimen dimensions on the fracture work of copy paper. (*) $a/w = 0.83$; (△) $a/w = 0.77$; (◆) $a/w = 0.67$.

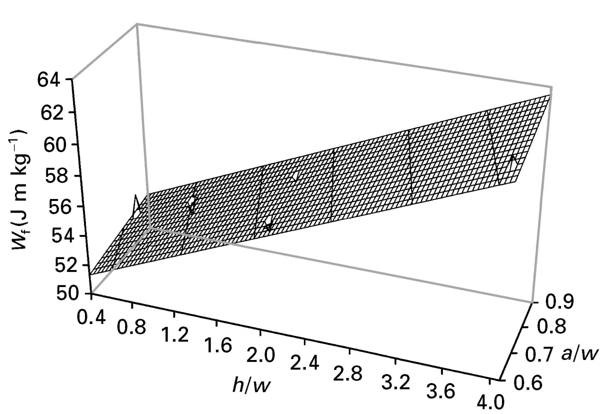


*Figure 6* External work needed to break sack paper specimens as a linear function of specimen dimensions.

the number of experiments is less than in the case of the copy paper, we find from Fig. 7 that the specific external work needed to break the specimen does not depend on specimen dimensions provided that the specimen is not longer than its width. In case the specimen is much longer than its width, the external work done to break the specimen increases with specimen length.

## 4. Discussion

The results suggest that with the present copy paper, plastic deformations surrounding the crack tip do not shield the crack tip enough to promote stable crack growth. Thus all the energy applied is not consumed by plastic deformations. Instead, the externally measured work consumed for breaking a copy paper specimen follows well a specimen-size dependency derived from linear elastic fracture mechanics. This indicates that the specific essential work of fracture of such a material cannot be measured as proposed by Cotterell and Reddel [3, cf. 9, 11, 12, 13]. Most machine-made information papers being more brittle than copy paper, this method of measurement is not applicable to them either.

On the other hand, pulp handsheets may be more tough and ductile than copy paper, at least in the case of well-beaten softwood pulp handsheets. With such materials, unbiased measurements can possibly be made. However, great care is needed, since even the work needed to break extremely tough and ductile sack paper depends on specimen dimensions if the specimens are considerably longer than their width.

## Acknowledgements
The authors are obliged to J. Rikala for experimental arrangements, C. Fellers for comments on the manuscript and H. Fullenwider for linguistic consultation.

## References
1. K. B. BROBERG, *Int. J. Fract.* **4** (1968) 11.
2. *Idem*., *J. Mech. Phys. Solids* **19** (1971) 407.
3. B. COTTERELL and J. K. REDDEL, *Int. J. Fract.* **13** (1977) 267.
4. Y.-W. MAI and B. COTTERELL, *Engng Fract. Mech.* **21** (1985) 123.
5. *Idem*., *Int. J. Fract.* **32** (1986) 105.
6. Y.-W. MAI and P. POWELL, *J. Polymer Sci. B* **29** (1991) 785.
7. Y.-W. MAI and B. COTTERELL, *J. Mater. Sci.* **15** (1980) 2296.
8. B. COTTERELL and Y.-W. MAI, in "Advances in Fracture Research" edited by D. Francois (Pergamon Press, Oxford, 1981) p. 1683.
9. R. S. SETH, A. G. ROBERTSON, Y.-W. MAI and J. D. HOFFMANN, *Tappi* **76** (1993) 109.
10. Y.-W. MAI, *Int. J. Mech. Sci.* **35** (1993) 995.
11. R. S. SETH, *Tappi* **78** (1995) 177.
12. Y.-W. MAI, H. HE, R. LEUNG and R. S. SETH, *Fracture Mechanics: ASTM STP* **26** (1995) 587.
13. R. S. SETH, *Tappi* **79** (1996) 170.
14. Y.-W. MAI and A. G. ATKINS, *Strain Analysis* **15** (1980) 63.
15. K. B. BROBERG, *J. Mech. Phys. Solids*, **23** (1975) 215.
16. C. E. TURNER, *Mater. Sci. Engng* **11** (1973) 275.
17. R. S. SETH and D. H. PAGE, *J. Mater. Sci.* **9** (1974) 1745.
18. Y.-W. MAI, *ibid.* **10** (1975) 1271.
19. T. L. ANDERSON, "Fracture Mechanics: Fundamentals and Applications" 2nd edn. (CRC Press, Boca Raton, FL, 1995).
20. K. IKONEN and K. KANTOLA, "Murtumismekaniikka" (Fracture Mechanics, in Finnish) (Otatieto Oy, 1986) p. 413.
21. D. P. ROOKE and D. J. CARTWRIGHT, "Compendium of Stress Intensity Factors" (HMSO, London, 1976).
22. H. TADA, P. C. PARIS and G. R. IRWIN, "The stress analysis of cracks handbook" (Del Research Corporation, Missouri, USA, 1973).

*Received 12 July 1996*
*and accepted 17 June 1997*